Boolean semantics for count nouns and mass nouns

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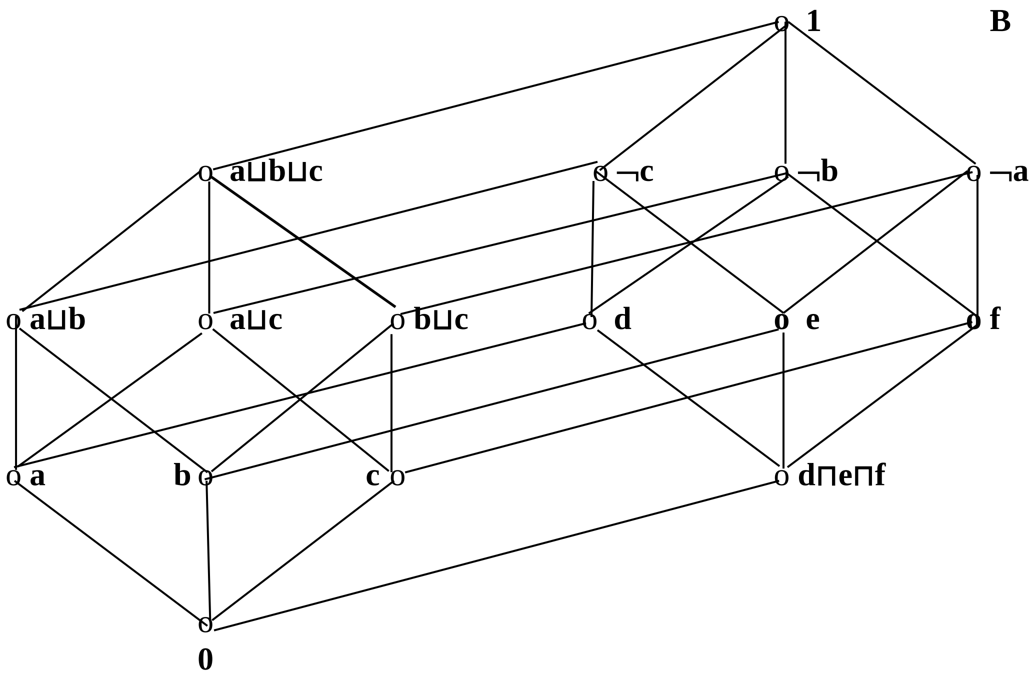
Lecture 1 Mountain Semantics

1.1. Boolean background

The structure **B** is a complete Boolea algebra**:**



-It is ordered by a **part-of relation ⊑**, such that e.g. **a** ⊑ **a⊔b⊔c**
-It is ordered by a **complete join (sum)** operation **⊔**, which maps any subset of B onto the join ⊔ (sum) of its element: the smallest element of B that all its elements are part of;

we write ⊔(a, b, c} = **a⊔b⊔c** for the sum of **a** and **b** and **c**.

-It is ordered by a **complete meet (overlap)** operation **⊓**, which maps any subset of B onto the meet ⊓ (overlap) of its element: the smallest element of B that is part of all its elements;

we write ⊓(**a**, **b**, **c**} = **d⊓e⊓f** for the overlap of **d** and **e** and **f**.

-⊔ and ⊓, as two place operations are distributive: **(a⊓b)⊔c** = **(a⊔c)⊓(b⊔c)**
-It has a minimal element **0**, null, **0 = ⊓B,** and a maximal element **1 = ⊔B**.
-It is ordered by a **complemetation operation ¬ (remainder)** such that:
for every a ∈ B: ¬a = ⊔{b ∈ B: a ⊓ b = 0}
The remainder of **a** (in B), ¬**a**, is the sum of all the elements of B whose overlap with **a** is null.
The **relative complement** of **a** in **b** (the remainder of **a** in **b**), b − a = ¬a ⊓ b.

Merelogical elements: Let $X \subseteq B$

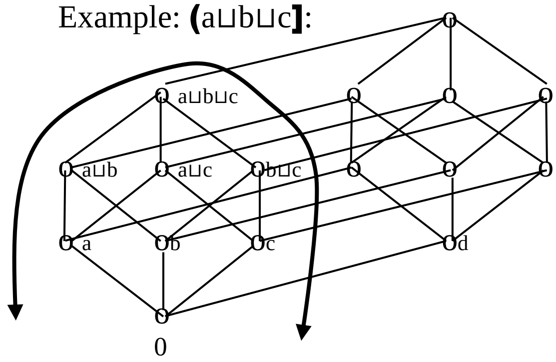
$X^+ = X - \{0\}$ The elements of B^+ we call *mereological* elements.

***Boolean part set*:** Let $x \in B$, $X \subseteq B$

$(x] = \{b \in B: b \sqsubseteq x\}$ The set of all Boolean parts of x

$(X] = (\sqcup X]$ The set of all Boolean parts of $\sqcup X$

Example: $(a \sqcup b \sqcup c]$:



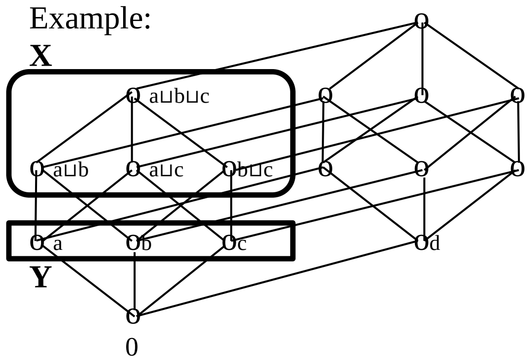
***Closure and generation under** sum* **⊔:** Let $X, Y \subseteq B$.

$*X = \{b \in B: \text{for some} Y \subseteq X: b = \sqcup Y\}$ The set of all sums of elements of X

Y *generates* X *under* ⊔ iff $X \subseteq *Y$ and $\sqcup Y = \sqcup X$

All elements of X are sums of elements of Y, and X and Y have the same supremum.

Example:



$*Y = Y \cup X \cup \{0\}$ and $\sqcup Y = \sqcup X$

hence Y generates X under ⊔

Atomicity: Let $a \in B$, $X \subseteq B$.

a is an X-*atom* iff $a \in X^+$ and for every $x \in X^+$: if $x \sqsubseteq a$ then $x = a$.

$ATOM_X$ is the set of X-atoms.

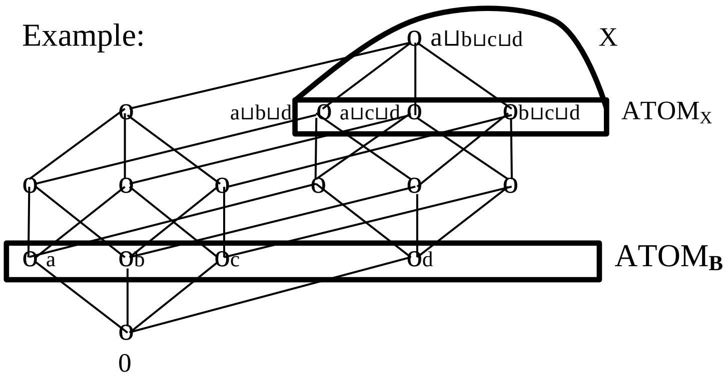
The set of mereological elements of X that have no proper parts that are also in X.

If 0 is not in X, then $ATOM_X$ is just the set of minimal elements in X.

Caveat: The notions defined here are the *generalizations* of the standard Boolean notions to arbitrary subsets of B. So they relate a subset X of B to **its** bottom, not to the bottom of B. You get the standard Boolean notions of set of atoms by taking X to be B:

a is an *atom in* **B** iff a is a B-atom.
$ATOM_B$ is the set of B-atoms

Example:



X is *atomic* iff for every $x \in X^+$ there is an $a \in ATOM_X$: $a \sqsubseteq x$
every element of X^+ has at least one part that is an X-atom.
X is *atomistic* iff for every $x \in X$ there is a set $A \subseteq ATOM_X$: $x = \sqcup A$
every element of X is the sum of X-atoms.
X is *atomless* iff $ATOM_X = Ø$
there are no X-atoms, i.e. X has no minimal mereological parts

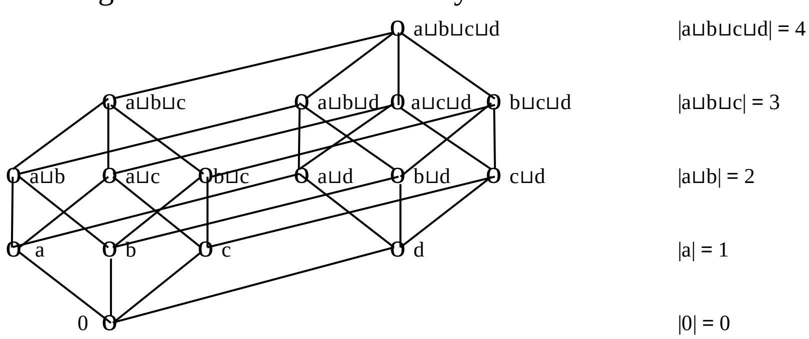
X-atomic parts:	Let $X \subseteq B$, $b \in X$
$ATOM_{X,b} = (b] \cap ATOM_X$	The set of X-atomic parts of b

Again the standard Boolean notions are the case where X is B:
B is *atomic/atomistic/atomless* iff B is *atomic/atomistic/atomless*

***cardinality** and cardinality function* Let $b \in B$

$\|b\| = \|ATOM_{B,b}\|$	$\|b\|$ is the cardinality of the set of b's atomic parts
card $= \lambda z.\|z\|$	The function that maps every element of b onto its cardinality

Ranking of **B** in terms of cardinality



Disjointness and overlap: Let $x,y \in B$, $X,Y \subseteq B$

We define overlap for mereological elements (elements in B^+)

x and y *overlap*: **overlap**(x,y) iff $x \sqcap y \in B^+$

Two elements overlap if they share a mereological, i.e. non-null, part.

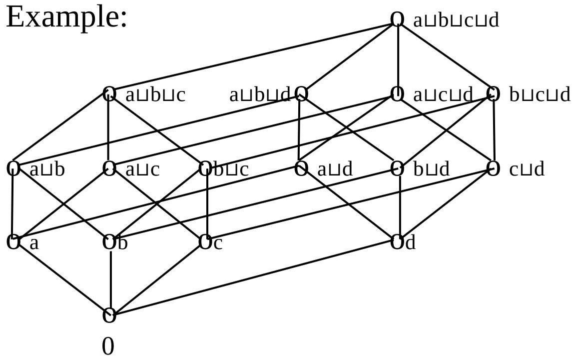
x and y are *disjoint*: **disjoint**(x,y) iff ¬**overlap**(x,y)

Two elements are disjoint if they do not overlap.

X *overlaps*: **overlap**(X) iff for some x, y ∈ X: **overlap**(x,y)

X is *disjoint*: **disjoint**(X) iff ¬**overlap**(X)

Example:



{a⊔b, c⊔d} is disjoint

{a⊔b, b⊔d} overlaps

{a,b,c,d} is disjoint

1.2. Mountain semantics for count nouns

Mountain semantics: Link 1983, 1984.
Central notions in the semantics of count nouns: atomicity and closure under sum (operation *).

1.2.1. Singularity and plurality

A *count domain* is a complete ***atomic*** Boolean algebra **B**

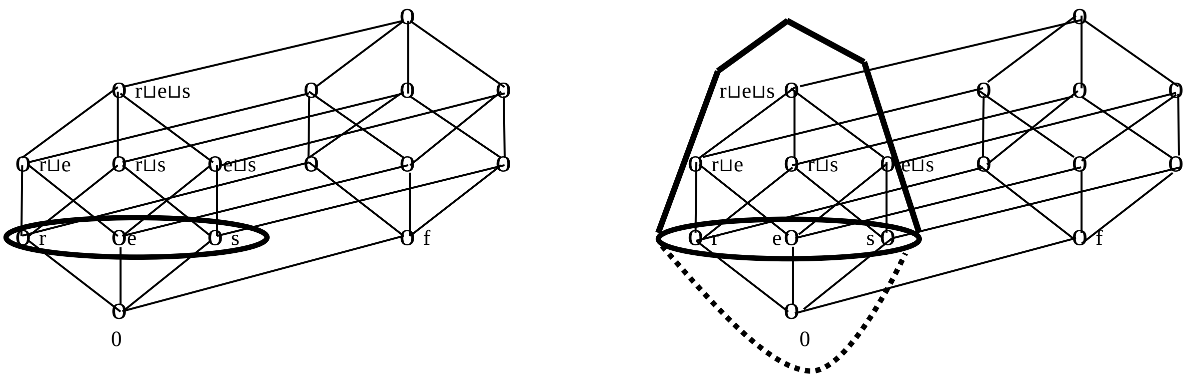
Singular nouns: *cat* $\rightarrow$ CAT_{wt} with $CAT_{wt} \subseteq ATOM_{\mathbf{B}}$

Singular nouns are interpreted as sets of atoms.

***Plural nouns*:** *cats* $\rightarrow$ $*CAT_{wt}$

Plural nouns are interpreted as the closure under sum of singular nouns.

Example: $ATOM_B$ = {ronya, emma, shunra, fido}



cat $\rightarrow$ CAT_{wt} = {r, e, s} $\subseteq$ $ATOM_B$ $*CAT_{wt}$ = {0, r, e, s, r⊔e, r⊔s, e⊔s, r⊔e⊔s}

Link 1983: semantic pluralization operation *
Mountain semantics: plural denotations are *mountains* rising up from the atomic seabed.

1.2.2 Counting in Mountain semantics

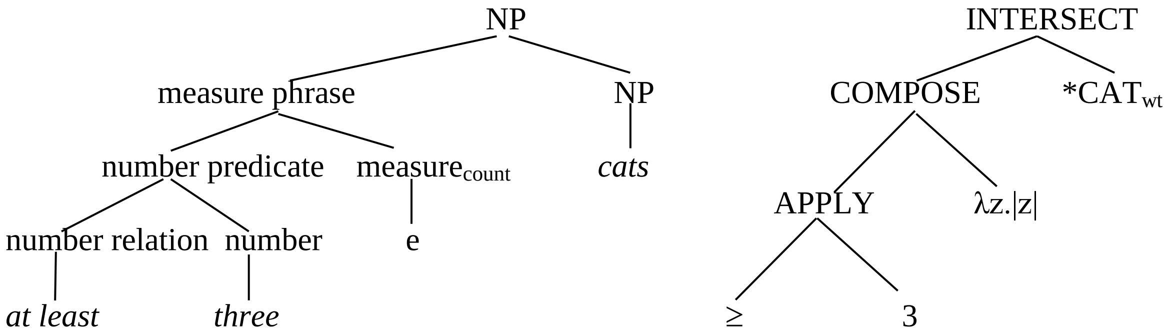
Numerical predicates. Based on Landman 2000, 2004
Guiding idea: The semantic composition takes place at the lowest available type.
(i.e. *three* is interpreted as 3, higher denotations are derived.)

Numbers: type n of numbers
three $\rightarrow$ 3 *eight* $\rightarrow$ 8

***Two place number relations*:** type of relations between numbers
at most $\rightarrow \leq$ *less than* $\rightarrow$ < *at least* $\rightarrow \geq$ *more than* $\rightarrow$ > *exactly* $\rightarrow$ =

***measure**$_{count}$* cardinality function
e $\rightarrow$ $\lambda x.|x|$

In this lecture: Syntax and semantics as in Landman 2004



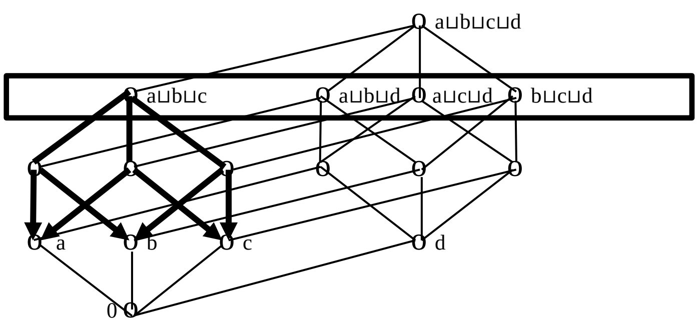
Number predicate: (≥(**3**)) = **λn.n ≥ 3**
The set of numbers from 3 up

Measure phrase: **λn.n ≥ 3 ∘ λz.|x|** **=** **λx.|x| ≥ 3**
The set of sums that have three atomic parts

NP: (λx.|x| ≥ 3) ∩ ***CAT$_{wt}$** = **λx.*CAT$_{wt}$(x) ∧ |x|≥3.**
The set of sums of cats that have three atomic parts.

For *three cats* I assume a null number relation with default interpretation =.
The semantics derives NP interpretations at the type <e,t>.
(For details about DP interpretations, see Landman 2005.)

So: *three* → λx.|x|=3

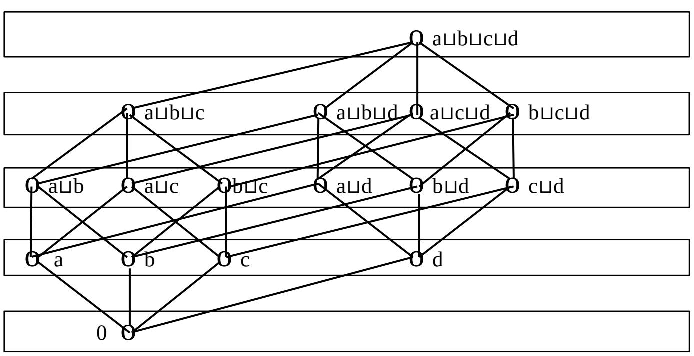


λx.|x|=3 = λx.|ATOM$_x$|=3

ATOM$_{a⊔b⊔c}$ = {a, b,c}, ATOM$_{a⊔b⊔d}$ = {a, b,d}
ATOM$_{a⊔c⊔d}$ = {a,c,d}, ATOM$_{b⊔c⊔d}$ = {b, c,d} .

λx.|x|=3 = {a⊔b⊔c, a⊔b⊔d, a⊔c⊔d, b⊔c⊔d}

In general: λx.|x|=n denotes the set of elements of **B** at height n:



λx.|x| = 4 = {a⊔b⊔c⊔d} 4

λx.|x| = 3 = {a⊔b⊔c, a⊔b⊔d, a⊔c⊔d, b⊔c⊔d}3

λx.|x| = 2 = {a⊔b, a⊔c, a⊔d, b⊔c, b⊔d, c⊔d} 2

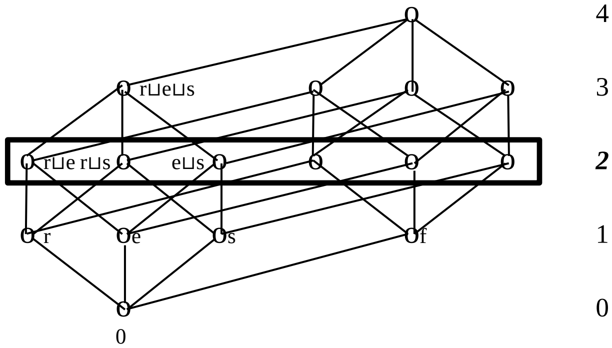
λx.|x| = 1 = {a, b, c, d} 1

λx.|x| = 0 = {0} 0

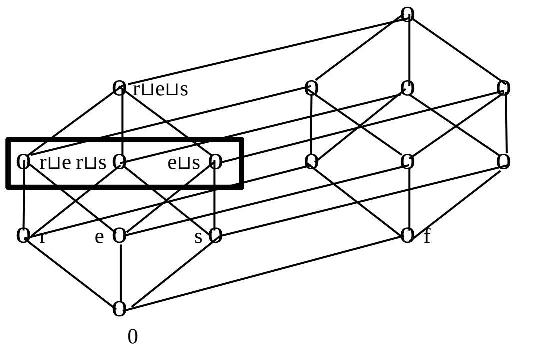
Upward and downward closure:

Neither up nor down:

Exactly two → $\lambda x.|x|=2$

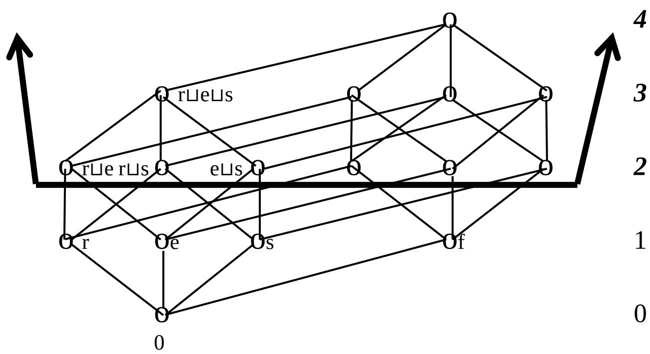


Exactly two cats → $\lambda x.{}^*CAT_{w,t}(x) \wedge |x|=2$

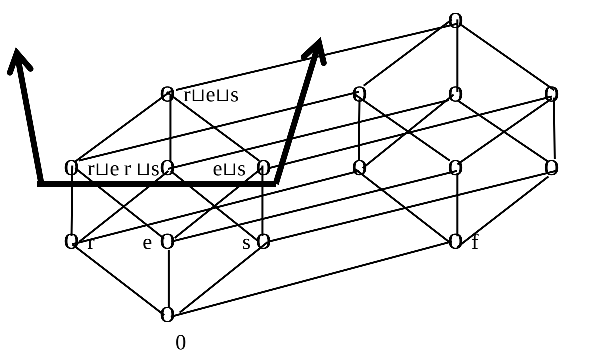


Up:

At least two → $\lambda x.|x|\geq 2$

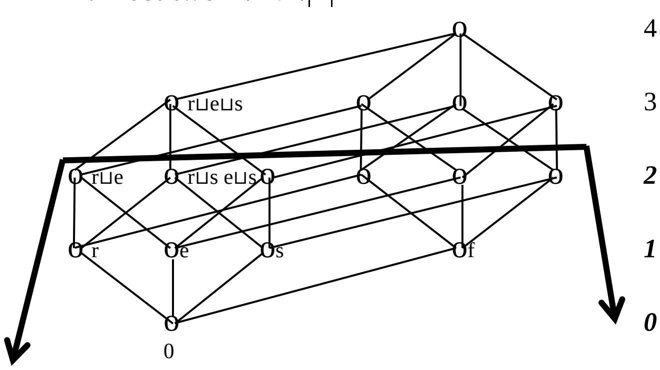


At least two cats → $\lambda x.{}^*CAT_{w,t}(x) \wedge |x|\geq 2$

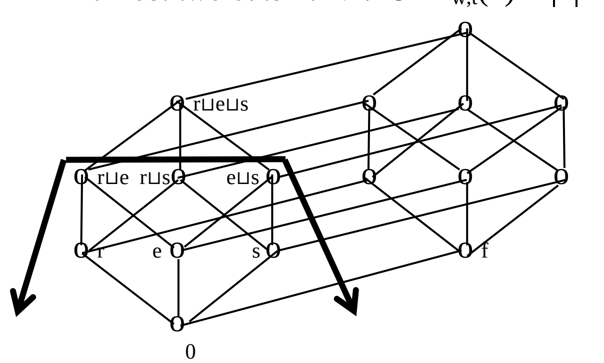


Down:

At most two → $\lambda x.|x|\leq 2$



At most two cats → $\lambda x.{}^*CAT_{w,t}(x) \wedge |x|\leq 2$



Visible in the picture: the polarity of the DPs is compositionally derived from the numerical relation.

Mountain semantics: Counting makes reference to atoms.

For now: ***at least three*** $\cap$ ***cats***
Later: mass/count presuppositions: infelicitous if the head noun isn't count.

1.2.3. Count comparison

(1) a. Most Dutch voted for Conchita.

Reading concerned: in this Contest more Dutch voted for Conchita than didn't.
(For other readings, see Hackl 2001.)

(1a) shows the need for contextual restriction:
In context, *Dutch* is restricted to: *Dutch that voted in the Eurovision Song Contest 2014*:

(1) b. Most Dutch that voted in the Eurovision Song Contest 2016 voted for Conchita.

$\textit{Dutch}_{\text{singular}} \rightarrow \text{DUTCH}_{\text{wt}} \subseteq \text{ATOM}_{\mathbf{B}}$
voted for Conchita $\rightarrow \lambda a.\text{VOTE}_{\text{wt}}(a,c) \subseteq \text{ATOM}_{\text{B}}$

Let $\mathcal{C}$ be a contextual variable. In the context we set:
$\mathcal{C} \rightarrow \text{ESC}_{2014} \subseteq \text{ATOM}_{\text{B}}$
The people that voted in ESC_{2014}

$\textit{Dutch}_{\text{plur}\mathcal{C}} \rightarrow {}^*(\text{DUTCH}_{\text{wt}} \cap \text{ESC}_{2014})$

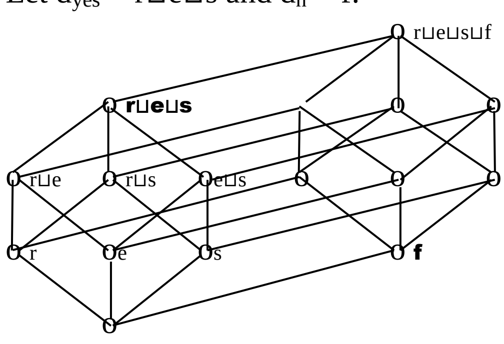
The context dependent semantics for *most* is:

$\textit{most} \rightarrow \lambda Q\lambda P.\ |\sqcup(Q \cap \mathcal{C}) \sqcap \sqcup P| > \sqcup(Q \cap \mathcal{C}) - \sqcup P|$

(3) $\textit{most}(\textit{Dutch}_{\text{plur}\mathcal{C}}, \textit{voted for Conchita}_{\text{plur}})$
$|\sqcup({}^*(\text{DUTCH}_{\text{wt}} \cap \text{ESC}_{2014})) \sqcap \sqcup({}^*\lambda a.\text{VOTE}_{\text{wt}}(a,c))| >$
$|\sqcup({}^*(\text{DUTCH}_{\text{wt}} \cap \text{ESC}_{2014})) - \sqcup({}^*\lambda a.\text{VOTE}_{\text{wt}}(a,c))|$

(3) compares: d_{yes} = the sum of all the Dutch that voted in ESC_{2014} and voted for Conchita
and: d_{no} = the sum of all the Dutch that voted in ESC_{2014} and didn't vote for Conchita,
and expresses that: the cardinality of the set of atomic parts of d_{yes} is bigger than that of the set of atomic parts of d_{no}.

Let $d_{yes} = r \sqcup e \sqcup s$ and $d_n = f$:



o

$|r \sqcup e \sqcup s| = 3$ $|f| = 1$ $3 > 1$, so (1a) is true.
Mountain semantics: count comparison makes reference to atoms.

1.2.4. Definite DPs

***Sum conjunction*:** *and* $\rightarrow \lambda y \lambda x.\ x \sqcup y$

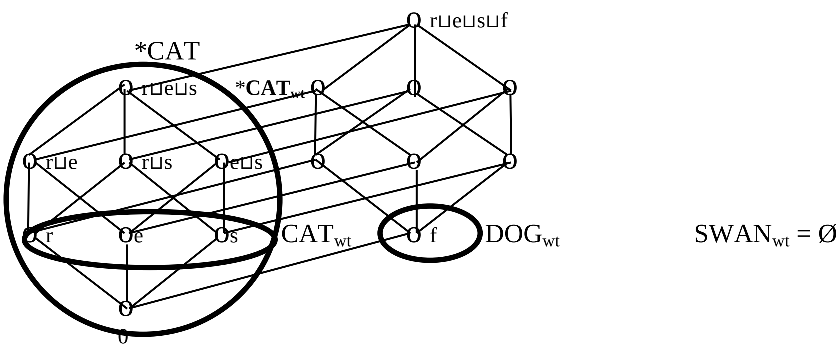
Ronya $\rightarrow$ r *Emma* $\rightarrow$ e *Shunra* $\rightarrow$ s r, e, s $\in ATOM_B$
Ronya and Emma and Shunra $\rightarrow r \sqcup e \sqcup s$

Definite article Sharvy 1980**:** *the* $\rightarrow \sigma$ presuppositional sum operation

$$\sigma = \lambda P. \begin{cases} \sqcup P & \text{if } \sqcup P \in P \\ \bot & \text{otherwise} \end{cases}$$

This analysis of the definite article was proposed in Sharvy 1980.
It is not yet in Link 1983, but added in Link 1984.
Assume the following noun denotations:

cat $\rightarrow CAT_{w,t} = \{r, e, s\}$
dog $\rightarrow DOG_{w,t} = \{f\}$ (fido)
swan $\rightarrow SWAN_{wt} = \emptyset$



Predictions:
[1] **a. *the cat* is undefined in wt**
the cat $\rightarrow \sigma(CAT_{wt})$ $\sigma(CAT_{wt}) = \bot$ because $\sqcup(\ \ CAT_{wt}) = r \sqcup e \sqcup s$ and $r \sqcup e \sqcup s \notin CAT_{wt}$
b. *the cats* denotes r⊔e⊔s in wt.
the cats $\rightarrow \sigma({*}CAT_{wt})$ $\sigma({*}CAT_{wt}) = r \sqcup e \sqcup s$
because $\sqcup({*}CAT_{wt}) = r \sqcup e \sqcup s$ and $r \sqcup e \sqcup s \in {*}CAT_{wt}$
c. *the two cats* is undefined in wt
$\sqcup(\lambda x.{*}CAT_{wt}(x) \wedge |x|=2) = r \sqcup e \sqcup s$ and $r \sqcup e \sqcup s \notin \lambda x.{*}CAT_{wt}(x) \wedge |x|=2$
d. *the more than two cats* denotes r⊔e⊔s in wt
$\sqcup(\lambda x.{*}CAT_{wt}(x) \wedge |x| \geq 2) = r \sqcup e \sqcup s$ and $r \sqcup e \sqcup s \in \lambda x.{*}CAT_{wt}(x) \wedge |x| \geq 2$

[2] ***the dog* denotes f in wt.**
the dog $\rightarrow \sigma(DOG_{wt})$ $\sigma(DOG_{wt}) = f$ because $\sqcup\{f\} = f$ and $f \in DOG_{wt}$

1.2.5 The pragmatics of the null object

Standard assumption:

Definiteness: For definite DP α:
$[\![\alpha]\!]_{wt} = \bot$ means that α is infelicitous in wt because of presupposition failure.

What if $[\![\alpha]\!]_{wt} = 0$? We find the following:

$SWAN_{wt} = Ø$

[3] **a. *the swan* is undefined in wt** $\bot$
the swan $\rightarrow \sigma(SWAN_{wt})$ $\sigma(CAT_{wt}) = \bot$ because $SWAN_{wt} = Ø$ and $\sqcup(Ø) = 0$ and $0 \notin Ø$
b. *the more than three swans* is undefined in wt $\bot$
the more than three swans $\rightarrow \sigma(\lambda x.{*}SWAN_{wt}(x) \wedge |x|>3)$
$\sigma(\lambda x.{*}SWAN_{wt}(x) \wedge |x|>3) = \bot$ because $\lambda x.{*}SWAN_{wt}(x) \wedge |x|>3 = Ø$ and $\sqcup(Ø) = 0$ and $0 \notin Ø$

Fact: Since $Ø \subseteq Ø$ and $\sqcup Ø = 0$, it follows by definition of * that $*Ø = \{0\}$.
Hence: $\boldsymbol{\sqcup\{*Ø\} = 0}$

[4] **a. *the swans* denotes 0 in wt.** **0**
the swans $\rightarrow \sigma({*}SWAN_{wt})$ $\sigma({*}SWAN_{wt}) = 0$ because $\sqcup{*}SWAN_{wt} = \sqcup{*}Ø = 0$ and $0 \in {*}Ø$
b. *the less than three swans* denotes 0 in wt. **0**
the less than three swans $\rightarrow \sigma(\lambda x.{*}SWAN_{wt}(x) \wedge |x|<3)$ =
$\sigma(\lambda x.{*}SWAN_{wt}(x) \wedge |x|<3) = 0$, because $\lambda x.{*}SWAN_{wt}(x) \wedge |x|<3 = \{0\}$ and $0 \in {*}Ø$

Landman 2004, 2010:
Triviality: For definite DP α:
$[\![\alpha]\!]_{wt} = 0$ means that α is **trivial** in wt, and this means that a statement $P(\alpha)$ will be ***trivially false or trivially true*** depending on the pragmatic choice of including or excluding 0 in the denotation of P.

I use in class example (4) to argue for Frege's analysis of quantifier *every*:

[I ran a crackpot lottery and stand in court. I know that I shouldn't commit perjury.
But I am better at Gricean pragmatics than the judge is, so I say:]
(4) a. Your honor, I swear that every person who, in the course of last year, presented me with a winning lottery ticket, has gotten his prize.
[I add, *sotte voce*, to you:]
b. Fortunately I was away all year on a polar expedition.

(4a), I argue with Frege, is not infelicitous because of a presupposition failure; (4a) is trivially true, since the denotation of the head noun *person who, in the course of last year, presented me with a winning lottery ticket* is empty. Hence, (4a) violates the maxim of Quantity (while, of course, I hope that the judge believes that it doesn't), but satisfies the Maxim of Quality (no perjury).

We look at the definite DPs: in the context where I was away all year.

(5) a. #Your honor, I swear that ***the one person*** who, in the course of last year, presented me with a winning lottery ticket, has gotten his prize.
b. #Your honor, I swear that ***the 5 persons*** who, in the course of last year, presented me with a winning lottery ticket, have gotten their prize.
c. #Your honor, I swear that ***the more than 30 persons*** who, in the course of last year, presented me with a winning lottery ticket, have gotten their prize.

All of these cases are clear cases of perjury due to the presupposition.

(6) a. ✓Your honor, I swear that ***the persons*** who, in the course of last year, presented me with a winning lottery ticket, have gotten their prize.
b. ✓Your horor, the books ought to tell you how many people came to me last year to claim their prize. I am sure it was less than 5. But I swear to you, your honor, that ***the less than five persons*** who, in the course of last year, presented me with a winning lottery ticket, have gotten their prize.

The cases in (6) pattern with (4): the non-emptyness of the noun interpretation is an implicature, rather than a presupposition, and the cases in (6) are quantity violations, rathern than perjury.

Not every English speaker is completely happy with NPs with complex nouns like (6b).
What is important here is the ***robustness of the contrast*** between the cases in (5) and in (6).

[Note that in (5a) I use *the one person*, rather than *the person*, because the latter allows a generic reading which, of course, patterns with the cases in (6).]

The same pattern is found in (7) and (8):

(7) a. In every family, ***the boy*** goes into the army.
b. In every family, ***the two boys*** go into the army
c. In every family, ***the more than three boys*** go into the army.

The examples in (7) ***presuppose*** that:

(7a) In every family there is one and only one boy
(7b) In every family there are exactly two boys
(7c) In every family there are more than three boys

Thus, if in one of the families there are no boys, all of (7a), (7b), (7c) are ***infelicitous***.

(8) a. In every family, ***the boys*** go into the army
b. In every family, ***the less than three boys*** go into the army

(8b) presupposes that in every family there aren't more than three boys. (Say, (8b) talks about the families that are left after that families with three or more boys have been excluded from the context.)

The crucial observation is that the cases in (8) do ***not presuppose*** that in every family there are boys.

Let φ_i = $FAMILY_{wt}(f_i) \wedge {}^*GOARMY_{wt}(\sigma({}^*\lambda a.BOY_{wt}(a) \wedge IN_{wt}(a,f_1))$

(8a) = $\forall f[FAMILY_{wt}(f) \rightarrow {}^*GOARMY_{wt}(\sigma({}^*\lambda a.BOY_{wt}(a) \wedge IN_{wt}(a,f))\ ] = \varphi_1 \wedge \ldots \wedge \varphi_n$

Look at φ_i and assume: $\lambda a.BOY_{wt}(a) \wedge IN_{wt}(a,f_i) = \varnothing$,
Then $\sigma({}^*\lambda a.BOY_{wt}(a) \wedge in_{wt}(a,f_i)) = 0$
and ${}^*GOARMY_{wt}(\sigma({}^*\lambda a.BOY_{wt}(a) \wedge in_{wt}(a,f_i))$ is trivially true (truthvalue 1).

Then: $\varphi_1 \wedge \ldots \wedge \varphi_{i-1} \wedge \boldsymbol{\varphi_i} \wedge \varphi_{i+1} \wedge \ldots \wedge \varphi_n$ =
$\varphi_1 \wedge \ldots \wedge \varphi_{i-1} \wedge \mathbf{1} \wedge \varphi_{i+1} \wedge \ldots \wedge \varphi_n$ =
$\varphi_1 \wedge \ldots \wedge \varphi_{i-1} \wedge \varphi_{i+1} \wedge \ldots \wedge \varphi_n$

Prediction: In (8a) *every family* quantifies over all families - including the ones without boys –
But the latter families f_i are irrelevant for the truth conditions of (8a),
because for them the condition φ_i holds **trivially.**

Conclusion: the Boolean theory of semantic plurality given here can make the distinction
between definite DPs whose denotation suffers from presupposition failure
and definite DPs whose denotation is trivial,
and this has interesting linguistic bite.

1.2.6 The distributive operator

(9) a. *The three cats* ate half a can of tuna.
b. *The three cats* ate half a can of tuna ***each***.

(9a): undetermined whether the cats ate half a can of tuna together,
or whether each of them ate that much tuna. (9b) distributor *each*: only the second reading.
Link 1983: *each* is interpreted as a distributive operator ^{D} that operates at the VP level:

$$^D = \lambda P \lambda x.\forall a \in ATOM_x\text{: } P(a)$$

Fact: For all $P \subseteq B$: $^DP = {}^*ATOM_{\sqcup P}$

Example: *cat* → $CAT_{wt} = \{r, e, s\}$
the three cats → $r \sqcup e \sqcup s$

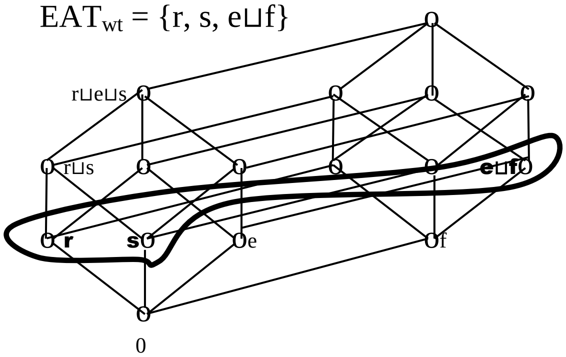
eat half a can of tuna → $EAT_{wt} \subseteq B$

eat half a can of tune each → $^DEAT_{wt} = \lambda x.\forall a \in ATOM_x\text{: } EAT_{wt}(a)$

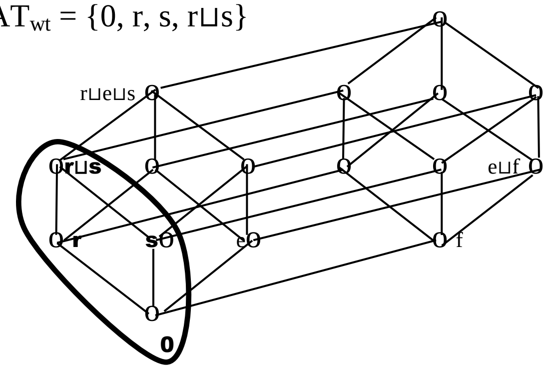
(9b) $\rightarrow \lambda x.\forall a \in ATOM_x: EAT_{wt}(a)(r{\sqcup}e{\sqcup}s) \ = \qquad \forall a \in ATOM_{r\sqcup e\sqcup s}: EAT_{wt}(a) \qquad =$
$EAT_{wt}(r) \wedge EAT_{wt}(e) \wedge EAT_{wt}(s)$

In a picture:
Let $EAT_{wt} = \{r, s, e{\sqcup}f\}$, the set of all objects in B that eat half a can of tuna.
${}^{D}EAT_{wt}$ is the set of all objects in B of which all the atomic parts eat half a can of tuna.

$EAT_{wt} = \{r, s, e{\sqcup}f\}$



${}^{D}EAT_{wt} = \{0, r, s, r{\sqcup}s\}$



We see that the semantics of distributive adverbial *each*, and more generally the distributive operator, make reference to atoms.
Mountain semantics: distribution makes reference to atoms.

In sum: the three phenomena - ***counting, cardinality comparison, and distributivity*** – are taken to be ***diagnostics for count nouns***. (Modification of this in lectures 2 and 3).

1.3. Mountain semantics for mass and count nouns

1.3.1. Boolean interpretation domains

Link 1983: Mass and count nouns are interpreted in distinct but linked domains.
Landman 1991's version:

A ***Boolean Interpretation Domain*** is a structure **I =** <**B**,**M**,**C**, ↓,↑>, where:

1. **B** is a complete Boolean algebra such that B = *(M ∪ C)
2. **M**, ***the mass domain***, is a complete Boolean algebra.
3. **C**, ***the count domain***, is a complete atomic Boolean algebra.
4. $0_{\mathbf{B}} = 0_{\mathbf{M}} = 0_{\mathbf{C}}$

5. ↓, ***the grinding function***, is a function from **C** into **M** such that:
   for every x ∈ C: $\downarrow c = \sqcup_{\mathbf{M}}\{\downarrow a\text{: } a \in ATOM_x\}$

   Grinding maps an individual onto the sum of its mass parts.

6. ↑, ***the portioning function***, is a one-one function from M into $ATOM_{\mathbf{C}} \cup \{0\}$ such that:
   ↑(0) = 0 and for every x ∈ M: ↓(↑(x) = x

   Portioning treats a mass object as an atomic count object.

Fact: ↓ is a join-homomorphism, a homomorphism that preserves 0, ⊑ and ⊔:

***The Boosk model*:** Link 1983

A ***boosk*** is a Boolean interpretation domain where ↑ is the identity function.

***The classical model*:** Landman 1991

A ***classical domain*** is a Boolean interpretation domain where M is atomless and $C^+ \cap M^+ = \emptyset$.

Let **B** be a complete Boolean algebra and Z ⊆ B. Let P: W×T → **pow**(B)

Count sets*:** Z is ***count iff **(**Z**]** is a complete ***atomic*** Boolean algebra or {0}
Count intensions*:** P is ***count iff for every wt: P_{wt} is count.
***Count noun phrases*:** NP is count iff NP is interpreted a *count* intension

Fact: If B is a classical domain then.
Count: If P: W×T → C then for **every** wt: P_{wt} is count
Mass: If P: W×T → M then for **every** wt: P_{wt} is only count if **(**Z**]** = {0}

The first part obviously holds in any Boolean Interpretation Domain.
But the second part holds, because in a Classical interpretation structure M is *atomless*, hence, for no x ∈ M^+ does it hold that **(**x**]** has the structure of a complete atomic Boolean algebra.

1.3.2 Semantics of lexical nouns

***Cumulativity and homogeneity*:** Let $X \subseteq \mathbf{B}$

X is *cumulative*, **cum**(X) iff X = *X — X is identical to its own closure under sum

X is *homogenous*, **hom**(X) iff $X = (\sqcup X]$ — X is identical to its own part set

***Facts*:** If **hom**(X) then **cum**(X)

If $X \subseteq ATOM_{\mathbf{B}}$, then **hom**(*X)

Intension P is **cumulative/homogenous** iff for every wt: P_{wt} is cumulative/homogenous

> **Krifka's assumption:**
> Let N be a lexical mass nouns N_{mass}
> $N_{mass} \rightarrow P$ where $P: W{\times}T \rightarrow \mathbf{M}$ and P is **cumulative**

- *and* and *the* are defined in the mass domain relative to sum operation $\sqcup_M$.

Example: mass noun *wine*:
wine → $WINE_{wt}$.

$WINE_{wt} \subseteq M$ and **cum**($WINE_{wt}$), which means that $WINE_{wt} = {*}WINE_{wt}$.
The latter means that $\sqcup WINE_{wt} \in WINE_{wt}$.
Hence:

the wine → $\sigma(WINE_{wt}) = \sqcup WINE_{wt}$
The sum of the wine

Counting, distribution and count comparison

In the classical domain there are no atoms in the mass domain.
This means that any operation that refers to atoms is undefined for the interpretations of mass nouns and mass DPs.

-**Counting phrases** are infelicitous with mass nouns: ***#one mud #three mud #three muds***
-Count comparison is infelicitous with mass nouns:

(10) a. Most cats purr
b. Most wine is red

(10a) means that there are more purring cats than non-purring cats.
(10b) does not mean that, say, there are more red wine particles than non-red wine particles.

-**Distribution** is infelicitous with mass nouns: ***#The wine is each burgondy red***

Mass comparison:

Let $\mathbb{R}^+ = \{r \in \mathbb{R}: r \geq 0\}$. Measure functions are functions $\boldsymbol{\mu}: W \times T \times M \to \mathbb{R}^+$
Let $\boldsymbol{\mu}$ be a contextually given measure function:

$\mathbf{most}(WINE_{wt},RED_{wt}) = \boldsymbol{\mu}_{wt}(\sqcup(WINE_{wt} \cap \mathcal{C}) \sqcap \sqcup RED_{wt}) > \boldsymbol{\mu}_{wt}(\sqcup(WINE_{wt} \cap \mathcal{C}) - \sqcup RED_{wt})$

A natural choice is $\boldsymbol{\mu}$ = **liter**
$\mathbf{most}(WINE_{wt},RED_{wt}) = \mathbf{liter}_{wt}(\sqcup(WINE_{wt} \cap \mathcal{C}) \sqcap \sqcup RED_{wt}) >$
$\mathbf{liter}_{wt}(\sqcup(WINE_{wt} \cap \mathcal{C}) - \sqcup RED_{wt})$

The volume in liters of the red wine ***is more than***
The volume in liters of what remains of the wine, when you take away the red wine.

Cumulativity
Krifka's assumption: Intension of lexical mass nouns is cumulative.
Hence: lexical mass nouns pattern with lexical plural nouns with respect to cumulativity (discussion in Krifka 1989):

If X is cumulative and $a, b \in X$ then $a \sqcup b \in X$

(11) a. If the animals in the house are cats and the animals in the garden are cats
then the animals in the house and the animals in the garden are cats
b. If the liquid in the glass is Marc de Bourgogne and the liquid in the bottle is Marc de Bourgogne, then the liquid in the glass and the liquid in the bottle is Marc de Bourgogne.

$*CAT_{wt}(\sigma(*AH_{wt})) \wedge *CAT_{wt}(\sigma(*AG_{wt})) \Rightarrow *CAT_{wt}(\sigma(*AH_{wt}) \sqcup \sigma(*AG_{wt}))$
$MB_{wt}(\sigma(LIQ_1)) \wedge MB_{wt}(\sigma(LIQ_2)) \Rightarrow MB_{wt}(\sigma(LIQ_1) \sqcup \sigma(LIQ_2))$

General evaluation:
-The theory of **count nouns:**
Singular-plural distinction in terms of atomicity and closure under sum.
This theory has been productive, succesful and influential:
Insightful analyses in different languages of phenomena like distributivity, collectivity, cumulativity, partitivity, pluractionality, etc., etc.

-The theory of **mass nouns:**
Mass-count distinction = atomic/atomless
Atomicity patterns with counting, count-comparison, distribution.
This theory is problematic.

-The **sortal distinction** between mass noun denotations (in **M**) and count noun denotations (in **C**).
This theory is problematic.

1.4. The sortal distinction between mass noun denotations and count noun denotations.

1.4.1 The gold paradox

The gold paradox is Link's main argument for sorting the domains for mass and count nouns.

[We have a ring of pure gold and nothing else.]

(12) ***The ring*** is pure gold and ***new***, but ***the gold making up the ring*** is ***old***.
$NEW_{wt}(\sigma(RING_{wt})) \wedge OLD_{wt}(\sigma(\lambda x.GOLD_{wt}(x) \wedge CONSTITUTE(x,\sigma(RING_{wt}))))$

Link's argument: if there is no sortal difference then the denotation of *the ring* and *the gold in the ring* is the same: $\sigma(RING_{wt}) = \sigma(\lambda x.GOLD_{wt}(x) \wedge CONSTITUTE(x,\sigma(RING_{wt}))$
Then (12) should be a contradiction. But it isn't.

Link: $\sigma(RING_{wt}) \in \mathbf{C}$ and $\sigma(\lambda x.GOLD_{wt}(x) \wedge CONSTITUTE(x,\sigma(RING_{wt})))$, $\in \mathbf{M}$,
and there is no contradiction.

Criticism of the argument (Rothstein 2011 and others):
You find this same paradox ***inside*** the mass domain and ***inside*** the count domain:

(13) a. ***The dry-stone wall*** is recent, but ***the stones making up the wall*** are ancient. [***count***]
b. The ***jewellery*** is new, but ***the gold it is made*** of is old. [***mass***]
c. ***The water*** is newly produced as the outcome of a chemical reaction, but ***the hydrogen and oxigen*** it is made of is, of course, very old. [***mass***]

A dry-stone wall is made of stones and nothing else.
Landman 1989: solve (13a) by sorting $ATOM_C$ into a set of **individual atoms** and **group atoms**.

Problem 1: Only in the context of examples like (2a) would you be tempted to let *the wall* denote a group atom rather than an individual atom.
One consequence of *the wall* as a group atom: hierarchy of groups of groups
Since Landman assumes that the collective reading of the **plural** DP *the walls* is also a group.
Problem 2: Similar arguments can be made for **any** singular DP: no individual atoms, only group atoms…

(13b) and (13c) show the same inside the mass domain.

1.4.2 The supremum argument

The supremum argument is based on Chiechia 1998.
Furniture is a mass noun,
Furniture items is a plural count noun, the pluralization of singular count noun *furniture item*.

Chierchia: the DPs *the furniture* and *the furniture items* have the same denotations:

$$\sqcup(FURNITURE_{wt}) = \sqcup(FURNITURE\text{-}ITEM_{wt})$$

This is suggested by examples like those in (14):

(14) a. We moved the ***furniture*** around.
b. We moved the ***furniture items*** around.
c. We moved the ***parts of the furniture*** around.

(14a) and (14b) are equivalent to each other; neither is equivalent to (14c).

The real situation is more complex (as the gold paradox already shows):
Chierchia's observation is correct to the extent that ***in default contexts*** corresponding mass and count DPs are naturally assumed to have the same denotation:

(15) a. In spring, Ronya's *hair* is falling out.
b. In spring, Ronya's *hairs* are falling out.

But, if I cut and recombine hairs of Ronya, and make very long strings, and turn this into a wig which I put on then (16a) is true, but (16b) is false:

(16) a. Ronya's *hair* looks ridiculous on Fred.
b. Ronya's *hairs* look ridiculous on Fred.

Nevertheless, even in this form, the supremum argument is important, and an argument ***against*** sorting: we want to be ***able*** to equate Ronya's hair and Ronya's hairs.

1.4.3. Portioning

This problem goes back to Landman 1991.

Coffee is a mass noun, *the coffee in the pot* and *the coffee in the cup* take their denotation in **M**:

Let $CP_{wt} = \lambda x.COFFEE_{wt}(x) \wedge IN_{wt}(x,\sigma(POT_{wt}))$
$CC_{wt} = \lambda x.COFFEE_{wt}(x) \wedge IN_{wt}(x,\sigma(CUP_{wt}))$

the coffee in the pot $\rightarrow \sigma(CP_{wt}) \in \mathbf{M}$
the coffee in the cup $\rightarrow \sigma(CC_{wt}) \in \mathbf{M}$

Hence, *the coffee in the pot and the coffee in the cup* takes its denotation in **M**:

the coffee in the pot and the coffee in the cup $\rightarrow \sigma(CP_{wt}) \sqcup \sigma(CC_{wt})$

This is a problem for (17):

[The laboratory confirmed:]
(17) The coffee in the pot and the coffee in the cup *were* ***each*** *spiked with strychnine*.

*were **each** spiked with strychnine* $\rightarrow$ $\lambda x.\forall a \in \text{ATOM}_x$: $\text{SPIKED}_{wt}(a)$

(17) $\rightarrow \lambda x.\forall a \in \text{ATOM}_x: \text{SPIKED}_{wt}(a)\ (\sigma(\text{CP}_{wt}) \sqcup \sigma(\text{CC}_{wt}))$

This should mean: $\forall a \in \mathbf{ATOM}_{\boldsymbol{\sigma}(\mathbf{CP}_{\mathbf{wt}}) \sqcup \boldsymbol{\sigma}(\mathbf{CC}_{\mathbf{wt}})}$: $\text{SPIKED}_{wt}(x)$

But $\sigma(\text{CP}_{wt}) \sqcup \sigma(\text{CC}_{wt}) \in \mathbf{M}$, hence atomicity and distributivity are not defined.

Solution (Landman 1991):
Apply ***portioning*** $\uparrow$ to the denotations of *the coffee in the pot* and *the coffee in the cup*, sum these, and apply the predicate to the result:

the coffee in the pot $\rightarrow \uparrow(\sigma(\text{CP}_{wt}))$ $\in \mathbf{ATOM_C}$ the coffee in the pot as a **portion** of coffee
the coffee in the cup $\rightarrow \uparrow(\sigma(\text{CC}_{wt}))$ $\in \mathbf{ATOM_C}$ the coffee in the cup as a **portion** of coffee

Hence, *the coffee in the pot and the coffee in the cup* takes its denotation in **C**:

the coffee in the pot and the coffee in the cup $\rightarrow \uparrow(\sigma(\text{CP}_{wt})) \sqcup \uparrow(\sigma(\text{CC}_{wt}))$

(17) $\rightarrow \lambda x.\forall a \in \text{ATOM}_x: \text{SPIKED}_{wt}(a)\ (\uparrow(\sigma(\text{CP}_{wt})) \sqcup \uparrow(\sigma(\text{CC}_{wt})))$
$= \text{SPIKED}_{wt}(\uparrow(\sigma(\text{CP}_{wt})) \wedge \text{SPIKED}_{wt}(\uparrow(\sigma(\text{CC}_{wt}))$

Problem:
Now we need to accept the coffee in the pot, $\sigma(\text{CP}_{wt})$, and $\uparrow(\sigma(\text{CP}_{wt})$, the coffee in the pot as a countable portion as two ontologically distinct entities. This distinction is forced by the theory.

Can't we just simplify the theory and get rid of the sorting?
This is what Iceberg semantics is about.
The answer is: yes.
But maintaining the advantages of the theory of count nouns requires work.

Tune in next time to Lecture 2 on Iceberg semantics.